

Outage Probability analysis of Power Domain MISO-NOMA in Rayleigh Fading Channel

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Abstract

The multiple access technique of the current wireless communication system depends on the Orthogonal Multiple Access (OMA) technique. The availability of orthogonal resources is limited in OMA and cannot support the increasing demand for a large number of users and efficient use of the spectrum in the coming years. The Non-Orthogonal Multiple Access (NOMA) is considered in the future release of 5G and Beyond 5G. In NOMA, multiple users can be dealt with in the same resource block at the same time, which allows NOMA to provide large connectivity, low latency, and high spectral efficiency. In this paper, the outage probability expression for the Multiple-Input Single-Output-Non-Orthogonal Multiple Access (MISO-NOMA) downlink system is derived by employing an Indefinite Quadratic Form (IQF) approach. Analytical results are compared with the Monte Carlo simulations which validate our theoretical derivations. Further, the performance of the MISO-NOMA system is assessed with different numbers of users in the system and different Signal-to-Noise Ratio (SNR) values.

Index Terms: Beyond 5G, Indefinite Quadratic Form, Multiple-Input Single-Output-Non-Orthogonal Multiple Access, Orthogonal Multiple Access, Outage Probability.

I. INTRODUCTION

Recently, the requirement for a high data rate has risen exponentially with the increase in the number of large data-consuming applications [1-4]. The multiple access techniques play an important role in handling this large amount of data by serving multiple users efficiently in the given bandwidth [5]. In wireless communication networks, the multiple access technique is considered the principal enabler and it advanced in every successive generation [6]. In 1G, FDMA was used along with the analog modulation, and TDMA was used in 2G GSM [7]. The dominant multiple access scheme was CDMA in 3G which was initially proposed by 'Qualcomm' [8]. The OFDMA was implemented in 4G networks to overcome the limitation of CDMA [9]. The multiple access techniques in previous generations are referred to as Orthogonal Multiple Access (OMA). There is very little interference among the adjacent resource block in all these OMA techniques as their resource blocks are divided orthogonally in code, frequency, or in the time domain which allows simple signal detection. The limited number of users is supported by the OMA technique due to less amount of usable orthogonal resource blocks which restrict the capacity and spectral efficiency in the present network. Therefore, to accommodate the enormous number of users, researchers proposed the Non-Orthogonal Multiple Access (NOMA) system [6]. As a result, the spectral efficiency can be increased by the use of NOMA and a huge number of data

can be transferred in the same bandwidth spectrum. The NOMA scheme is mainly divided into two key categories NOMA in the power domain and the code domain [10]. In a single orthogonal resource block, the fundamental of NOMA is applied to get spectral efficiency by utilizing the power domain multiplexing which leads to the power domain NOMA. In this power domain, several users are dealt with in the same OFDMA subcarrier or time slot and power domain multiple access can be implemented by giving dissimilar power levels to each user [11], and [12]. The NOMA can be united with other techniques to increase spectral efficiency by exploiting the advantage of both technologies. The CR-NOMA is the integration of cognitive radio and NOMA scheme which results in the reduction of transmission delay in secondary users by the simultaneous connection of multiple secondary users in the system [13]. The combination of NOMA with millimeter-wave further enhances the spectral efficiency as 'mmWave' provides enormous bandwidth for communication [14]. The NOMA scheme can also combine with the MIMO technique by exploiting the advantages of the MIMO technique resulting in significant improvement as compared to MIMO-OMA [15]. The authors in [4], investigated the efficient energy design of the MISO-NOMA system without having the perfect knowledge of the channel at BS. Clusters are formed by the clustering algorithm and resource sharing is done by employing the NOMA scheme. The power constraint



beamforming optimization problem with per-antenna is investigated in [16] for downlink MISO-NOMA.

Basically, MISO-NOMA is the combination of ‘Multiple-Input Single-Output’ and ‘Non-Orthogonal Multiple Access’ techniques. The MISO-NOMA exploits the advantage of both techniques, i.e., minimizing errors, optimizing data speed, and large connectivity. In this paper, we acquired the expression of outage probability for the power domain downlink MISO-NOMA system by employing the approach Indefinite Quadratic Form (IQF) presented in [17].

The remaining part of the paper is structured as below:

The downlink MISO-NOMA system model is explained in Section II and the SINR expression is derived in Section III. The expression of outage probability is derived in Section IV. The outcomes of outage probability analysis are determined in Section V. In the end, Section VI concludes the paper.

II. DOWNLINK MISO-NOMA SYSTEM MODEL

The downlink communication system consists of the MISO-NOMA technique. There are ‘ l ’ single antenna users and BS consists of ‘ N ’ antenna elements and it transmits signals by superposing multiple user’s signals with different power coefficients. Successive Interference Cancellation (SIC) is performed successively to recover the desired signal of the user. The allocation of power to each user depends upon the channel condition of each user. A lesser amount of power is assigned to the user with a good channel and a large power is given to the user with a bad channel. The user consists of the highest power to recover its signal without performing the SIC. The SIC process is needed to perform by the remaining users in the systems [18].

The base station transmits the signal ‘ s ’ as written below:

$$s = \sum_{l=1}^L \sqrt{a_l P_s} x_l \quad (1)$$

In eq. (1), the transmitted information of the ‘ i^{th} ’ user is represented as ‘ x_i ’ having unit energy.

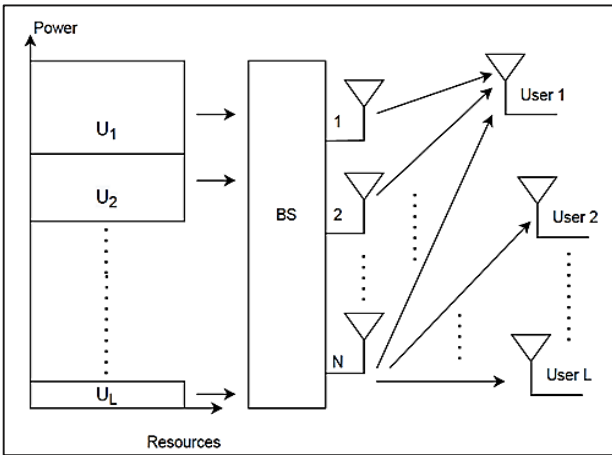


Figure 1: Downlink Power Domain MISO-NOMA System

Further, the transmission power of BS is represented as ‘ P_s ’ and the assigned power coefficient of the ‘ l^{th} ’ user is represented as:

$$a_l \text{ 'subject to } \sum_{l=1}^L a_l = 1.$$

In this system, we considered the power coefficient to be $a_1 \geq a_2 \geq \dots \geq a_L$. Therefore, the channel gains are supposed to be in the order of $|h_1|^2 \leq |h_2|^2 \leq \dots |h_L|^2$ [2].

The ‘ l^{th} ’ user channel coefficient can be written as ‘ h_l ’ and the received signal for the ‘ l^{th} ’ user can be given as follows:

$$y_l = h_l s + n_l = h_l \sum_{i=1}^L \sqrt{a_i P_s} x_i + n_l \quad (2)$$

where the complex additive Gaussian noise is expressed as ‘ n_l ’ with variance ‘ σ_l^2 ’ and ‘0’ mean. i.e., $n_l \sim CN(0, \sigma_l^2)$.

III. DERIVATION OF SINR EXPRESSION

The SINR expression can be derived from eq. (2), desired user signal power divided by the interference power and the noise variance.

In this case, l^{th} user is considered as desired user power:

$$SINR_l = \frac{\gamma a_l \|h_l\|^2}{\gamma \|h_l\|^2 \sum_{i=j+1}^L a_i + \sigma_l^2} \quad (3)$$

In eq. (3), the ‘ γ ’ is denoted by $\gamma = P_s$ and the lower limit of $\sum_{i=j+1}^L a_i$ is $j+1$ in the denominator, it is because successive interference cancellation will be performed for the values $j \leq l$.

By employing the quadratic form approach eq. (3) can be expressed as:

$$SINR_l = \frac{\|\bar{h}_l\|_A^2}{\|\bar{h}_l\|_{B+\sigma_l^2}^2} \quad (4)$$

Where;

$$A = R^{\frac{1}{2}} \begin{bmatrix} \gamma a_l & 0 & 0 \\ 0 & \gamma a_l & 0 \\ 0 & 0 & \gamma a_l \end{bmatrix} R^{\frac{H}{2}} \quad (5)$$

$$B = R^{\frac{1}{2}} \begin{bmatrix} \gamma \sum_{i=j+1}^L a_i & 0 & 0 \\ 0 & \gamma \sum_{i=j+1}^L a_i & 0 \\ 0 & 0 & \gamma \sum_{i=j+1}^L a_i \end{bmatrix} R^{\frac{H}{2}} \quad (6)$$

Where;

R is the correlation matrix of the Channel.

IV. DERIVATION OF OUTAGE PROBABILITY EXPRESSION

By using the eq. (4), and ‘ γ_{th} ’ considered as a given threshold, $\Pr(SINR_l < \gamma_{th})$ is the CDF for the probability outage of ‘ l^{th} ’ user available in the cell for notation simplicity it is written as $F_1(\gamma_{th})$ and expressed as follows:

$$F_l(\gamma_{th}) = \Pr \left(\frac{\|\bar{\mathbf{h}}_l\|_A^2}{\|\bar{\mathbf{h}}_l\|_{B+\sigma_l^2}^2} < \gamma_{th} \right) \quad (7)$$

By simplifying eq. (7) we get:

$$F_l(\gamma_{th}) = \Pr \left(\gamma_{th} \sigma_l^2 + \|\bar{\mathbf{h}}_l\|_{B\gamma_{th}-A}^2 > 0 \right) \quad (8)$$

The CDF in eq. (8) can be written in the form of eq. (9) by employing the definition of 'u(.)' which is a unit step function:

$$F_l(\gamma_{th}) = \int_{-\infty}^{\infty} p(\bar{\mathbf{h}}_l) u \left(\sigma_l^2 \gamma_{th} + \|\bar{\mathbf{h}}_l\|_{B\gamma_{th}-A}^2 \right) d\bar{\mathbf{h}}_l \quad (9)$$

The integration in eq. (9) is on the complete $\mathbf{h}_l$ -plane and therefore, the existence of 'u(.)' supports the analysis in the work. The unit step function can be written in Fourier representation in [3] is given below:

$$u(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{x(j\omega+\beta)}}{j\omega+\beta} d\omega; \quad \beta > 0 \quad (10)$$

By substituting the PDF of channel vector $p(\bar{\mathbf{h}}_l) = \frac{1}{\pi^T} e^{-\|\bar{\mathbf{h}}_l\|^2}$ and after simplification we get:

$$F_l(\gamma_{th}) = \frac{1}{2\pi^{T+1}} \int_{-\infty}^{\infty} \frac{e^{\sigma_l^2 \gamma_{th} (j\omega+\beta)}}{j\omega+\beta} \times \int_{-\infty}^{\infty} e^{-\|\bar{\mathbf{h}}_l\|_{I+(A-B\gamma_{th})(j\omega+\beta)}^2} d\bar{\mathbf{h}}_l d\omega \quad (11)$$

The matrix P is defined where $\mathbf{P} = \mathbf{A} - \mathbf{B}\gamma_{th}$ and after performing its eigenvalue decomposition we obtained $\mathbf{P} = \mathbf{U}_p \mathbf{\Lambda} \mathbf{U}_p^H$. The channel vector $\bar{\mathbf{h}}_l$ can be transformed with the relation of $\tilde{\mathbf{h}}_l = \mathbf{U}_p^H \bar{\mathbf{h}}_l$.

The weight matrix of the quadratic norm can be obtained in diagonal norm by this transformation in eq. (11) and can be written as:

$$\begin{aligned} \frac{1}{\pi^T} \int_{-\infty}^{\infty} e^{-\|\tilde{\mathbf{h}}_l\|_{I+\mathbf{\Lambda}(j\omega+\beta)}^2} d\tilde{\mathbf{h}}_l &= \frac{1}{|I + \mathbf{\Lambda}(j\omega + \beta)|} \\ &= \frac{1}{\prod_{t=1}^T |1 + \lambda_t(j\omega + \beta)|} \end{aligned} \quad (12)$$

Where;

λ_t represents the tth eigenvalue of P. Thus, the T-dimensional integral in eq. (11) is changed to a corresponding 1-dimensional form written as:

$$F_l(\gamma_{th}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{\sigma_l^2 \gamma_{th} (j\omega+\beta)}}{(j\omega+\beta) \prod_{t=1}^T (1+\lambda_t(j\omega+\beta))} d\omega \quad (13)$$

Assuming that the eigenvalues are distinct. Thus, the partial fraction expansion can be used to expand the inner denominator term which will result in:

$$\begin{aligned} &\frac{1}{(j\omega + \beta) \prod_{t=1}^T (1 + \lambda_t(j\omega + \beta))} \\ &= \frac{1}{j\omega + \beta} + \sum_{t=1}^T \frac{\alpha_t}{(1 + \lambda_t(j\omega + \beta))^t} \end{aligned} \quad (14)$$

Where;

α_t is the coefficient of partial fraction expansion which can be found to be:

$$\alpha_t = \frac{-\lambda_t}{\prod_{i=1, i \neq t}^T \left(1 - \frac{\lambda_i}{\lambda_t}\right)} \quad (15)$$

Next, employing the following rule from the residue theory [4]:

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{j\omega p}}{(a + j\omega)^s} d\omega = \frac{\text{sign}^s(a)}{\Gamma(s)} (p)^{s-1} e^{-ap} u(ap) \quad (16)$$

Where;

$\Gamma(\cdot)$ and $\text{sign}(\cdot)$ shows the gamma function and signum function, respectively.

Thus, the CDF $F_l(\gamma_{th})$ for the ' l^{th} ' user can be shown to be:

$$\begin{aligned} F_l(\gamma_{th}) &= u(\sigma_l^2 \gamma_{th}) \\ &- \sum_{t=1}^T \frac{\lambda_t^T}{\prod_{i=1, i \neq t}^T (\lambda_t - \lambda_i) |\lambda_t|} \frac{1}{|\lambda_t|} e^{-\frac{\sigma_l^2 \gamma_{th}}{\lambda_t}} u\left(\frac{\sigma_l^2 \gamma_{th}}{\lambda_t}\right) \end{aligned} \quad (17)$$

Finally, using the definition of outage probability (i.e., from the certain threshold value the probability that the SINR is less), the outage probability of the ' l^{th} ' user closed form in the MISO-NOMA downlink system is found to be:

$$\begin{aligned} P_{out,l}(\gamma_{th}) &= u(\sigma_l^2 \gamma_{th}) \\ &- \sum_{t=1}^T \frac{\lambda_t^T}{\prod_{i=1, i \neq t}^T (\lambda_t - \lambda_i) |\lambda_t|} \frac{1}{|\lambda_t|} e^{-\frac{\sigma_l^2 \gamma_{th}}{\lambda_t}} u\left(\frac{\sigma_l^2 \gamma_{th}}{\lambda_t}\right) \end{aligned} \quad (18)$$

V. RESULTS AND DISCUSSION

In this section, the probability of an outage for the MISO-NOMA downlink system is analyzed by considering the different scenarios such as different SNR values and the different numbers of users in the systems.

In figure II, the outage probability for simulation and analytical results are compared. It is clear from the curves that the simulation and analytical outage probabilities are showing the same behavior for all threshold values from 0 to 3. This result validates the derived expression of outage probability in eq. (18).

The effect of SNR on outage probability is investigated in figure III. In this analysis, three SNR values are considered SNR 0 dB, SNR 10 dB, and SNR 20 dB. The SNR values are considered based on the channel condition. The low SNR value depicts the noisy channel and the high SNR value shows the less noisy channel. The curves show that the outage probability for a low SNR is higher than the outage probability of a high SNR value. This outcome shows that in poor channel conditions, the probability of outage is significantly high in the MISO-NOMA downlink system as compared to good channel conditions.

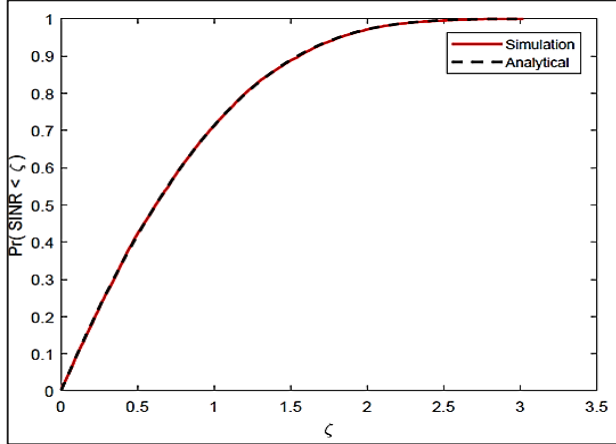


Figure II: Outage Probability Simulation and Analytical Comparison

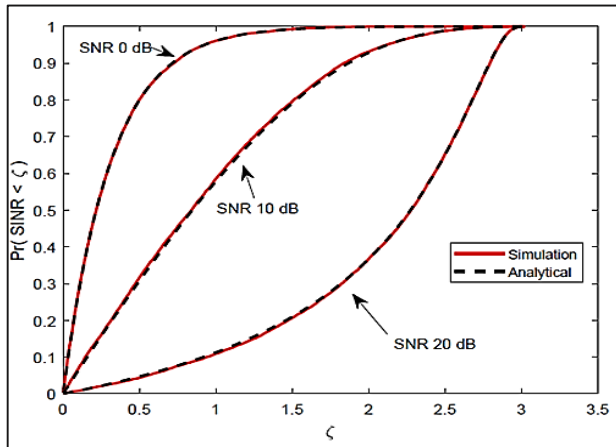


Figure III: Effect of SNR on the Outage of Probability

In figure IV, the effect of the number of users on the outage probability for downlink MISO-NOMA is analyzed.

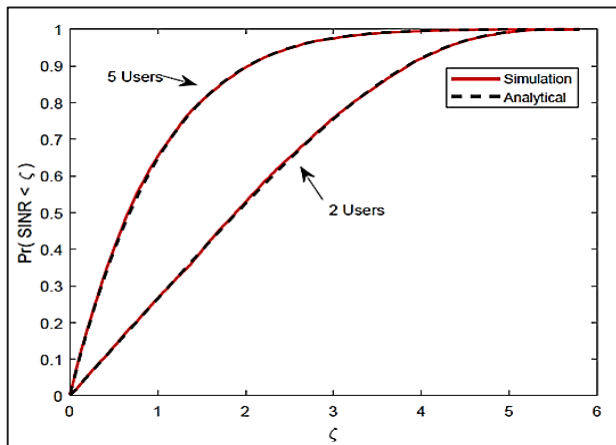


Figure IV: Effect of Number of Users on Outage Probability

In this analysis, we considered two users and five user scenarios in the MISO-NOMA downlink system.

It is evident from the curves that the outage probability is higher in 5 user scenario as compared to the 3 users in the system. It is due to the increase in interference in the system by the rise in the number of users.

In this system model, other than the desired users are considered as interference. Thus, an increase in the number of users will decrease the overall system performance.

VI. CONCLUSION

The OMA technique is not suitable to accommodate the massive number of users and demand for a high data rate. The researchers considered the NOMA technique to support the massive connectivity and be part of the future generation.

In NOMA in the power domain, different level of power is allocated to different users in the same resource block which increases the spectral efficiency. The expression of outage probability is acquired for the MISO-NOMA downlink system by employing the IQF approach. The analytical and simulation results validate the derived expression of outage probability.

Further, the effect of SNR and the effect of the number of users in the systems are analyzed. The results show that the outage probability is low in good channel conditions with a high SNR value as compared to the poor channel condition. The number of users also affects the system performance as an increase in the number of users increases the probability of an outage and results in a reduction in the total system performance.

The studied system is based on fixed power allocation and optimum power allocation will increase the performance of the system.

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Authors Contributions

The contribution of the Authors are as follows:

Muhammad Ahsan Shaikh's contribution to this study was the concept, derivation, and implementation of outage probability, under the supervision and guidance of Dr. Anwaar Manzar. The methodology was proposed by Dr. Muhammad Moinuddin. Engr. Sadiq Ur Rehman compiled the manuscript. Engr. Halar Mustafa has done the proofreading and revision process of the manuscript.

Conflict of Interest

There is no conflict of interest between all the authors.

Data Availability Statement

The testing data is available in this paper.

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References

- [1] Ahmed, A., Elsaraf, Z., Khan, F. A., & Ahmed, Q. Z. (2021). Cooperative Non-Orthogonal Multiple Access for Beyond 5G Networks. *IEEE Open Journal of the Communications Society*, 2, 990–999.
- [2] Al-Naffouri, T. Y., & Hassibi, B. (2009, June). On the distribution of indefinite quadratic forms in Gaussian random variables. In *2009 IEEE International Symposium on Information Theory* (pp. 1744–1748). IEEE.
- [3] Akhtar, S. (2009). 2G-5G networks: Evolution of technologies, standards, and deployment. *Encyclopedia of Multimedia Technology and Networking*, 522–532.
- [4] Alavi, F., Cumanan, K., Ding, Z., & Burr, A. G. (2018). Beamforming techniques for nonorthogonal multiple access in 5G cellular networks. *IEEE Transactions on Vehicular Technology*, 67(10), 9474–9487.
- [5] Aldababsa, M., Toka, M., Gökçeli, S., Kurt, G. K., & Kucur, O. (2018). A tutorial on nonorthogonal multiple access for 5G and beyond. *Wireless communications and mobile computing*, 2018.
- [6] Al-Naffouri, T. Y., Moinuddin, M., Ajeeb, N., Hassibi, B., & Moustakas, A. L. (2016). On the Distribution of Indefinite Quadratic Forms in Gaussian Random Variables. *IEEE Transactions on Communications*, 64(1), 153–165.
- [7] Benjebbour, A., Saito, Y., Kishiyama, Y., Li, A., Harada, A., & Nakamura, T. (2013). Concept and practical considerations of non-orthogonal multiple access (NOMA) for future radio access. In *2013 International Symposium on Intelligent Signal Processing and Communication Systems*, (770–774). IEEE
- [8] Cai, Y., Qin, Z., Cui, F., Li, G. Y., & McCann, J. A. (2018). Modulation and Multiple Access for 5G Networks. *IEEE Communications Surveys & Tutorials*, 20(1), 629–646.
- [9] Chandra, K., Marciano, A. S., Mumtaz, S., Prasad, R. V., & Christiansen, H. L. (2018). Unveiling Capacity Gains in Ultradense Networks: Using mm-Wave NOMA. *IEEE Vehicular Technology Magazine*, 13(2), 75–83.
- [10] Choi, J. (2016). Power Allocation for Max-Sum Rate and Max-Min Rate Proportional Fairness in NOMA. *IEEE Communications Letters*, 20(10), 2055–2058.
- [11] Dai, L., Wang, B., Yuan, Y., Han, S., Chih-lin, I., & Wang, Z. (2015). Non-orthogonal multiple access for 5G: Solutions, challenges, opportunities, and future research trends. *IEEE Communications Magazine*, 53(9), 74–81.
- [12] Ding, Z., Lei, X., Karagiannidis, G. K., Schober, R., Yuan, J., & Bhargava, V. K. (2017). A Survey on Non-Orthogonal Multiple Access for 5G Networks: Research Challenges and Future Trends. *IEEE Journal on Selected Areas in Communications*, 35(10), 2181–2195.
- [13] Dinh-Thuan Do, A., & Lee, B. M. (2020). NOMA in Cooperative Underlay Cognitive Radio Networks Under Imperfect SIC. *IEEE Access*, 8, 86180–86195.
- [14] Ghosh, J., Ra, I.-H., Singh, S., Haci, H., Al-Utaibi, K. A., & Sait, S. M. (2022). On the comparison of optimal NOMA and OMA in a paradigm shift of emerging technologies. 17. *IEEE Access*, 10, 11616–11632.
- [15] Gilhousen, K. S., Jacobs, I. M., Padovani, R., Viterbi, A. J., Weaver, L. A., & Wheatley, C. E. (1991). On the capacity of a cellular CDMA system. *IEEE Transactions on Vehicular Technology*, 40(2), 303–312.
- [16] Huang, Yongwei; Zhou, Longtao (2021). MISO NOMA downlink beamforming optimization with per-antenna power constraints. *Signal Processing*, 179, 107828.
- [17] Gradshteyn, I. S., Ryzhik, I. M., & Jeffrey, A. (2007). Table of integrals, series, and products (7th ed). *Academic Press*
- [18] Islam, S. M. R., Avazov, N., Dobre, O. A., & Kwak, K. (2017). Power-Domain Non-Orthogonal Multiple Access (NOMA) in 5G Systems: Potentials and Challenges. *IEEE Communications Surveys & Tutorials*, 19(2), 721–742.