Performance Evaluation of MRMED Algorithm by Monitoring BER

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***Abstract*— In this research work, we have developed a communication system (transmitter/receiver) to control Peak to Average Power Ratio (PAPR) with small Bit Error Rate (BER) for a 4G system called Multi-Code Code Division Multiple Access (MC-CDMA). Proposed communication system works on Modified Reed Muller Encoded Data (MRMED) string. In MRMED data is first encoded with Reed Muller (RM) code. Thereafter, encoded RM message is 'XOR'ed with optimal binary string, which results in lower PAPR. A well-known fact is that, BER is the best performance measurement tool for a communication system. To check the integrity of our communication system, we have run the simulation for monitoring BER using MRMED sequence. Simulation work conducted, with multipath Rayleigh fading, Minimum Shift Keying (MSK) modulation and several orders of RM codes. Our results show that implementing MRMED sequences of the suggested MC-CDMA communication structure, returns noticeable lower BER. For instance, in case of RM (1,4), that has error improvement proficiency of 3 (three) errors , returns BER = 8.2x10−5 adopting MSK, at Signal-to-Noise Ratio (SNR) = 12dB. Similarly, for RM (2,3), which has error improvement efficiency of 0 (zero) error and shows distinct BER of 4.9x10−4 at SNR=12dB. In addition to using simulation for checking BER performance of our communication system, we have also shown in our results that, as the error improvement capability of different RM codes surges, correspondingly we get a lower BER.**

***Index Terms*— Bit Error Rate (BER), Reed Muller (RM) codes, Multi-Code Code Division Multiple Access (MC-CDMA), Peak to Average Power Ratio (PAPR), Signal-to-Noise Ratio (SNR).**

I. INTRODUCTION

The Multi-Code Code Division Multiple Access (MC-CDMA) worthiness includes backward compatibility and providing high data rates. In MC-CDMA number of channels is added and this addition of channels result in unreasonable Peak to Average Power Ratio (PAPR), that creates distortion. Therefore, a need arise for costly, Power Amplifier (PA) or use some other techniques like coding to reduce PAPR [1-4].

There are several methods to make Bit Error Rate (BER) lower, are the main focus of this research work. In order to make BER lower, two researchers introduces blind channel estimation for downlink Wideband Code Division Multiple Access (W-CDMA) system that applied, Chaotic codes and Walsh Hadamard codes for spreading data bits of end users. To check system integrity of blind channel estimation they have employed BER for measuring their system performance [5].

Some investigators showed that employing multi-code i.e., Direct-Sequence Code Division Multiple Access (DS-CDMA) channelization scheme improves BER in multipath fading channel [6].

The Characteristic Function (CF) method and indivisible law approach was developed by a group of researchers. This approach asserts that fading channel has deterministic transmission ability to reduce BER, provided that transmitter keeps transmitting at constant data rate [7].

Investigators proposed Kasami code to reduce BER under (AWGN and Rayleigh fading) [8].

Some researchers implemented MC-MC-CDMA system, to minimize BER [9].

Our idea to reduce BER for MC-CDMA communication system is depend on Modified Reed Muller Encoded Data (MRMED) algorithm. The objective of the MRMED algorithm (briefly described in **Section III**) is to reduce PAPR. In the said algorithm RM code is used to encode data. Nevertheless, with multiple channel assignments RM code does not work. Therefore, we modified the RM encoded data sequence.

For high data rates, different channels are given to a mobieuser, who demands for large data rates. These additions of channels result in the high peaks. A recognized measure for high peaks is known as PAPR that is the ratio of peak power to its average power.

Mathematically, PAPR for an un-coded data (ud) or message with its corresponding signal S (t) can be written as:

$$\mathrm{PAPR(ud)} = (1/x)\max|\mathrm{ud} * \mathrm{WHT_x}|^2 \qquad (1)$$

Where:
WHT = Walsh Hadamard Transform
and
x = Length of WHT

WHT is a square matrix and its cross-correlation between two rows or channels is zero, which avoids mutual interference between channels (users). We know that high peaks make distortion in the PA, consequently this distortion give rise to BER.

Our proposed MRMED algorithm makes the PAPR lower and we obtain a minimum BER for our MC-CDMA transmitter/receiver system. The MC-CDMA transmitter and

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receiver are shown in Fig. 1.
The MRMED sequence algorithm with monitoring BER briefly outlined in **Section III** of the proposed methodology. Pursuance of a communication structure is best scaled by its BER [10-12]. The ratio of erroneous bits (received) to the overall bits transferred over a wireless channel is known as BER.

$$\mathrm{BER} = (\mathrm{T\_Errors})/(\mathrm{T\_bits}) \qquad (2)$$

Where:
BER = Bit Error Rate
T_Errors = Overall number of bits in error received
and
T_Bits = Overall number of bits transmitted.

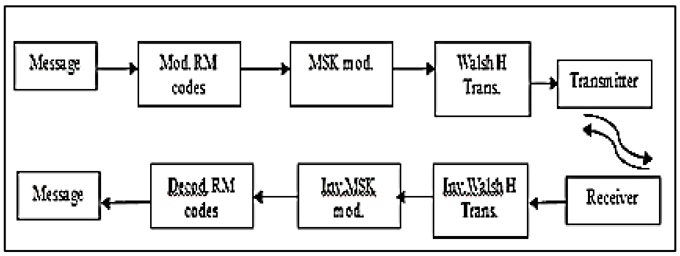

Fig. 1: MC-CDMA Transmitter/Receiver

Before, we start our discussion about related work and proposed methodology for monitoring BER. A brief introduction to RM codes, Rayleigh multipath fading, multipath delay spread, Minimum Shift Keying (MSK) modulation and Signal-to-Noise Ratio (SNR) transmission bandwidth is given as follows:

A. RM codes

They are among the earliest and great-appreciated, families of codes. RM codes are simple to construct and decrypt [13]. Binary RM codes are defined for m and r, $0 \leq r \leq m$, there is a binary r^{th} order RM code RM(r, m) with the given below parameters:

$$\left[n = 2^m, k = 1 + \binom{m}{1} + \cdots + \binom{m}{r}, d = 2^{m-r}\right] \qquad (3)$$

Here:
n = express the length of the data,
m = variable,
k = dimension of code subspace and
d = the minimum distance

Large order of RM codes is recurrently built from the lower order and usually make basis for making another codes. Application of RM (1, m) is in the very noisy channel, range finding, scrambling, and synchronizing [14].

B. Multipath Rayleigh Fading Model

In wireless communication multipath Rayleigh fading channels are helpful standard to replicate the actual problem. Multipath Rayleigh blurring may incorporate multipath diffusing impacts, time scattering, and Doppler shifts that come from related movement between the transmitter and receiver of a communication mechanism. Rayleigh distribution applied for fading process, in case there is no Line Of Sight (LOS) between the transmitter and receiver. The Rayleigh fading originated from multipath reception of various forms of signals [15], [16].

C. Multipath Delay Spread

Multipath arises when signal reached at the mobile unit directly from the transmitter or discursively through obstacles like building, trees, mountains etc. For example, an impulse transmitted at some time t; assuming that many reflected paths are present, a mobile unit at some distance away would find series of pulses with some delay. Now we would consider the outcome of this delay spread in frequency domain.
Consider, two multi-paths having the same amplitude C, and τ is a delayed path, which is relative to another multipath. The obtained signal$(t)$is given below [17]:

$$(t) = (t) + CS(t - \tau) \qquad (4)$$

To convert the above equation from time domain to frequency domain, we take the Fourier transform.

$$X(f) = CS(f) + CS(f)e^{-j2\pi f\tau} \qquad (5)$$

By rearranging

$$X(f) = CS(f)[1 + e^{-j2\pi f\tau}]$$
$$X(f) = CS(f)H(f)$$

Where:
H (f) is the transfer function of the wireless channel, which transforms the signal **CS (f)**.
H (f) can be expressed as:

$$H(f) = 1 + e^{-j2\pi f\tau}$$
$$H(f) = e^{-j2\pi f(\tau/2)}\left[e^{j2\pi f(\tau/2)} + e^{-j2\pi f(\tau/2)}\right]$$
$$H(f) = 2e^{-j2\pi f(\tau/2)}cos\big(2\pi f(\tau/2)\big)$$

Hence, the magnitude spectrum $|(f)|$ is given as follows:

$$|(f)| = 2cos\big(2\pi f(\tau/2)\big) \qquad (6)$$

D. MSK Modulation

The binary digits when shift from 0 to 1 or reversed, produces unwanted transition, which conceive undesired signals that have sidebands spanning out from the carrier and therefore produce interference between the nearby channels. This issue was rectified in Offset Quadrature Phase Shift Keying (OQPSK). Nevertheless, a 90° phase difference was existed in OQPSK. However, it was solved and the developed modulation scheme is recognized as Continuous Phase Modulation (CPM). The MSK is one of the modulation theories [18]. As there are no phase breaks in MSK signal, hence MSK portray abridged power spectrum than binary Phase Shift Keying (PSK) [19].
The remaining paper is classified as follows:
In **Section II**, we discuss relevant work. **Section III** is about proposed methodology, which also covers our contribution. **Section IV** gives simulation results and discussion and **Section V** ends up with a conclusion and open issues.

E. SNR and Transmission Bandwidth

The role of various orders of RM codes over SNR and transmission bandwidth is mentioned in **Section VII** of simulation results and discussion. Results revealed that minimal SNR, turnout exceptional BER for various forms of RM codes. Nevertheless, for a minimum BER (good quality of signal), price for bandwidth expansion (ciphering of data) should not be a matter of consideration against quality of signal.

The Power 'P' of a signal is related to attribute of transmission. Increasing P reduces effect of noise in the channel. Anyhow, reduced SNR is important for communication, because it is proportional to the power P of a signal; therefore, SNR and bandwidth are correlative.

Hence, a little rise in channel bandwidth gives the advantage of minimum power.

II. RELATED WORK

For BER computation two researchers introduced blind channel estimation for downlink Wideband Code Division Multiple Access (W-CDMA) system that applied Chaotic codes and Walsh code. Regarding, Chaotic codes and Walsh code, at SNR = 12dB they calculated BER of 1 × 10-4 and 13 × 10-4 respectively [20].

Some investigators presented noticeable BER can be accomplished by employing Multi-code DS/CDMA approach in the multi-path fading channel [21].

The Characteristic Function (CF) method and infinitely indivisible law has been evolved by some investigators. This affirm that the independent and identically distributed (i.i.d) fading channels has deterministic transmission potentiality (zero error probability) [22].

While investigators computed the BER achievement of DS-CDMA for Kasami code under AWGN and Rayleigh fading channel for different modulation schemes. For AWGN channel, it was found that BER was, minimum [23].

More approaches to lower BER such as MC-MC-CDMA system, clearly defeat single-code multicarrier CDMA (MC-CDMA) and single-carrier multi-code CDMA with respect to error probability and user capacity in frequency selective multipath Rayleigh fading channel [24].

Following comparison table, i.e., Table I show all above-mentioned related work to reduce BER.

Table I: Proposed Methods for Minimizing BER

Proposed Methods to Minimize BER	BER/ SNR	AWGN Channel / Multipath Rayleigh and Rician Fading Channels
Chaotic Codes and Walsh Codes [5]	1 × 10^-4 and 13 × 10^-4 / 12dB	Rayleigh and Rician Fading Channels
Multicode DS - CDMA [6]	Average Bit Error Prob.=10^-6 / 20dB	Rayleigh Fading Channel
Characteristic Function (CF) Approach and Infinitely Indivisible Law[7]	Zero Error Probability. Note; If Transmitter Maintains Constant Data Rate	Nakagami-m Fading Channel
DS-CDMA for Kasami [8]	10^-0.30 / 28dB	AWGN and Rayleigh Fading Channel
MC-MC-CDMA System [9]	10^-5 / 30dB	Frequency Selective Fading
MRMED Algorithm[4] RM(1,4) and RM(1,5)	8.2x10^−5 and 1.982x10^-05 / 12dB	Multipath Rayleigh Fading Channel

III. PROPOSED TECHNIQUE

A. *Contribution*

Following current objectives are part of the above research as specified in **Section II** i.e., related work:

- Built theoretical MCCDMA transmitter/receiver model, based on MRMED sequence
- Our model is straightforward as compared to related work of **Section II**
- Explore various orders of RM codes
- MSK modulation
- Noticeable BER is obtained: For RM(1,5), BER = 1.982x10^-05, at SNR=12dB
- Multipath Rayleigh fading

Table IV: Performance Parameters for 1st and 2nd Orders RM Code

RM Codes	Code Rate	Minimum Distance	Error Improvement
RM(2,3)	7/8	2	0
RM(1,3)	4/8	4	1
RM(2,4)	11/16	4	1
RM(1,4)	5/16	8	3
RM(2,5)	16/32	8	3
RM(1,5)	6/32	16	7

MCCDMA transmitter and receiver are shown previously in Fig. 1. First message is encoded using RM code and then it goes to modified RM codes (mod. RM codes) algorithm, where ciphered message is XORed with optimum binary progression. Thereafter, modified RM encoded message deliver to MSK modulation (MSK mod.), Walsh Hadamard Transform (Walsh HTrans) and lastly to Transmitter (Trans). Channel between the Transmitter and Receiver is multipath Rayleigh fading. At the receiver reverse process is performed to extract the original data. BER is computed on decrypting side of the receiver.

B. *Outline of the Algorithm*

- Let message '***emi***' represent i^{th}. encoded message of RM code of orders 0, 1 and 2
- Each bit of the encoded message '***emi***' is 'XOR'ed with finest binary sequence '***B***'
- The dimension of '***B***' should be equivalent to the length of encoded message '***emi***'
- Exhaustive search to find best '***B***' is performed
- Our algorithm selects best '***B***' bit to 1 that yields minimal PAPR

The pseudo code of MRMED sequences are given [25].

IV. SIMULATION RESULTS AND DISCUSSION

For modified and original RM codes of 1^{st} and 2^{nd} order, simulation is implemented using criterion as depicted in Table II over 4-path Rayleigh fading, until overall errors attained to 100 bits. In addition, 500 kHz of signal bandwidth (Bs) is assumed.

Table II: Simulation

Parameter	Specification
Multiple Access Technique	MC-CDMA
Modulation	MSK
Rayleigh Multipath Fading	4-Path
Fading	Frequency Selective
Total Errors for this Simulation	100 bits (at each SNR of 0,3,6,9 and 12dB)
Coherence Bandwidth (Bc)	54.64khz
Signal Bandwidth (Bs)	500khz
root mean square (rms) delay (σ_τ)	3.66μs
Message Length in Bits	4,5,6,7,11, 16 Corresponds to the following RM Codes:- RM(1,3), RM(1,4), RM(1,5) RM(2,3), RM(2,4), RM(2,5)

Since coherence bandwidth (Bc), which is less than signal bandwidth (Bs). Therefore, fading is selective. The power delay profile used in this simulation is also shown in Table III. Normally, the lagging of the first path is fixed to zero and for the following paths, a delay of 1 μs correlates to a 300m variation in path length. The delay profile as exhibited in Table III, we speculate a 3 dB loss in average power for whole 4μs of path delay [26] .

Table III: 4-Path Power Delay Profile

Delay, μs	Power (dB)
0.0	0
4	-3
8	-6
12	-9

We have calculated BER for 1st and 2nd order of modified RM(1,5), RM(2,3) codes and original RM(1,5),RM(2,3) codes as shown in Fig. 2. This Fig. 2 shows that modified RM codes of both orders, results in lower BER compare to that of original RM codes. The reduction in lower BER of modified RM codes proves that using modified RM codes outperforms the original RM codes in terms of lowering BER.

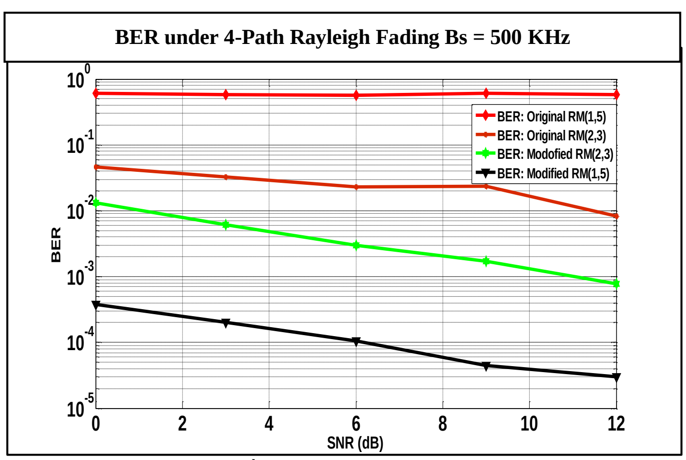

Fig. 2: BER for 1st and 2nd Orders of Original and Modified RM Codes

Table IV presents an assessment for 1st and 2nd order of RM codes [27] . Consider Fig.2, which follows Table IV. In other words as the improvement in error of RM codes rises, a BER of modified RM codes decreases. For instance, in case of RM(2,3) and RM(1,5), here RM(1,5) has a lower BER compare with RM(2,3) , whereas RM(1,5) can improve7 errors, while RM(2,3) makes no improvement in error. However, the lower BER of any RM codes mentioned in Fig.2 is only possible if we use modified RM codes.

Fig.2, shows BER for RM (2,3) is 4.97x10^-04 at SNR=12dB and Fig. 3, shows BER for RM(2,3) is 2.7x10^-05 at SNR 24dB . Although, RM (2,3) makes no improvement in error. But, it has prominent BER at SNR = 24dB.

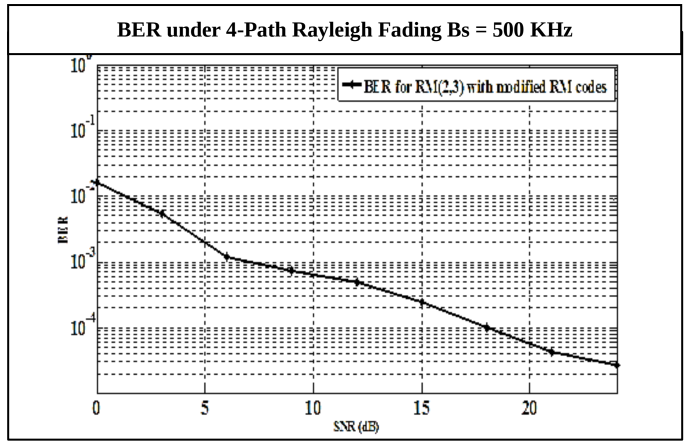

Fig. 3: BER for RM (2, 3) with Modified RM Codes

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RM(1,4)	5/16	8	3
RM(2,5)	16/32	8	3
RM(1,5)	6/32	16	7

V. CONCLUSION

Although, original (classical) RM codes have the built in error correction capability, but it may not be possible to reduce the BER without using MRMED sequence. Because, when it comes to assign different channels to the requirement of a mobile user, for higher data rates than it might not be possible to use classical RM codes. Original (classical) RM code would help for power reduction only, if we assign single channel to a user. Hence, combination of MRMED and error correction capability of RM codes for multiple channels provides a lower BER.

The BER for RM (1,5) is 1.982x10^-05 at SNR=12dB. However, at higher values of SNR, we could expect a very distinct BER for RM (1,5). RM (1,m) could be beneficial in very noisy channel. It is further concluded that MSK modulation also helps in lowering BER.

There are some open issues: Try to see the effect of MRMED with Offset Quadrature Phase Shift Keying (OQPSK); also, compare the BER of binary PSK with OQPSK; furthermore, can we build a transmitter or receiver or both using Field Programmable Gate Array (FPGA) kit?

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